UNIVERSITY OF CRAIOVA
FACULTY OF MECHANICS

CONTRIBUTIONS TO THE STUDY OF VIBRATIONS INFLUENCE UPON THE STRESS AND STRAINS OF BAR-TYPE LINKS

- Summary of the PhD Thesis -

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SUMMARY

Research of recent years regarding bar-type links vibrations gained an increasingly theoretical and applied importance because knowing the vibrations propagation and the dynamic effects of strains produced during working depending on loads, geometric and mechanical features of bars led to the finding of advantageous technical solutions in design and implementation of mechanisms which have to operate at high speeds or mechanisms whose links positions have to be accurate.

In the first chapter it is justified the importance of the theme and the evolution of models for bar-type links is presented, starting with Bernoulli-Euler, Love, Kecs and Timoshenko models and finishing with the numerical software GraHyb 2, presented at the 9th Conference of Dynamic Systems from Lodz, Poland, in 2007, a software for vibrations analysis of mechanical systems, subjected to simultaneous kinematic and dynamic excitations.

Here the finite elements method is presented as an important method for studying links with a finite number of degrees of freedom and three analyses based on it are described: an analysis of stress and strains, a dynamic analysis of flexible mechanisms and a dynamic analysis of elements with fast motions.

Then, there are presented some of the mathematical models for the vibrations of the bar-type links, considered continuous media, encountered in the technical literature speaking about the evolution of these models such as:

1. One-dimensional model for bending vibrations of the elastic bars solved by the wave propagation method.
2. Model for vibrations analysis of Timoshenko bars solved by Cebisev pseudo-spectral method.
3. Model for vibrations analysis of bars with continuous mass and held concentrated masses.
4. Model for modal decomposing analysis in order to study mixed limit problems.

In the following chapter there are presented the mathematical models in displacements for the vibrations of bar-type links with linear elastic behaviour. In order to obtain them, it is started from the equations of motion in displacements, for a straight link, linearly elastic, in roto-translation motion, where the total mass m_t is considered concentrated in "n" points and the method of the influence coefficients is applied.

For a straight, linearly elastic link, model of continuous medium in planar roto-translation motion, the equations of motion in displacements, in the coupled version, are obtained using Hamilton's principle from elasto-dynamics, by neglecting the cutting forces influence.

The mathematical models found for the planar roto-translation motions can be written using two differential matrix form operators $[L_0]$ and $[L_1]$, by grouping the terms of coupling between the longitudinal and transverse vibrations and the terms which gives the quality of models varying in time. By neglecting the term which includes differential matrix form operator $[L_1]$, it is obtained the decoupled, linear model with constant coefficients, in the first approximation.

Then, there are presented the mathematical models in displacements for the vibrations of straight bar-type links with linear viscoelastic behaviour for mechanical models with a finite number of degrees of freedom and the models of continuous media type in planar motion, namely the form of mathematical models with equations with partial derivatives, mathematical models in real time.

Mathematical models for bars with viscoelastic behaviour are obtained from classical linear equations of elasto-dynamics. Based on elasto-viscoelastic analogies stated by Alfrey and Lee, it is applied the unilateral Laplace transform with respect to time to these equations.

In the case of equations of motion obtained with Hamilton's principle, it is applied the same method, substituting Young's modulus of elasticity E with its Laplace transform $\tilde{E}(s)$.

In the following chapter there are presented accurate and iterative methods for solving the mathematical models presented for straight bars-type links with linear elastic behaviour.

Solving these mathematical models is done using Laplace transform and Fourier transform finite in sine or cosine in the terms of boundary conditions specific for technical applications. There are obtained algebraic systems where the unknowns are the displacements in their Laplace and Fourier images. The longitudinal and transversal displacements fields for bar-type links with linear elastic behaviour are obtained by reversing the integral transforms.

For mechanical models with a finite number of degrees of freedom, Laplace transform with respect to time is applied to the system of differential equations representing the mathematical model of the motion. This gives an algebraic system of equations, the unknowns being the Laplace images of the displacements, system which is solved elementary. The displacements fields are obtained by reversing the Laplace transform.

For the mathematical model of the first approximation for free vibrations, which is a decoupled model, linear and with constant coefficients, it is applied the unilateral Laplace transform with respect to time and then, to the first equation it is applied the Fourier transform in finite cosine and to the second equation the Fourier transform in finite sinus. It is obtained a decoupled algebraic system, where the unknowns are the displacements in their Laplace and Fourier images in the cosine, respectively in the sine.

It takes into account the boundary conditions which allowed the application of the two Fourier transforms to the original functions and to their Laplace images respectively. Then, there are reversed the Laplace and Fourier transforms and it is obtained the solution in the first approximation $\{u^{(1)}(x,t)\}$.

Now the vector $[L_1]\{u^{(1)}\}$can be determined in a first approximation with the found $\{u^{(1)}(x,t)\}$ and, if it is introduced in the general equation, the mathematical model in the second approximation is obtained. Its resolution with the help of the integral transforms gives the solution in the second approximation. The iterative process continues and the mathematical model in the "j"-th approximation is obtained. The solution in the "j"-th approximation is obtained in the same way and the process of successive approximations continue until the difference between two consecutive solutions is less than $\varepsilon > 0$ and sufficiently small, depending on the required accuracy of the calculus.

In the 5-th Chapter the accurate and the iterative methods for solving the mathematical models are described for straight bar-type links with linear viscoelastic behaviour.

Solving these models is done by applying to them the integral Fourier transforms in finite sine or cosine. Then, there are solved the resulted algebraic system of equations and the unknowns $\tilde{u}_i^*(n,s)$ where i = 1, ..., 3, for spatial motion and i = 1, 2, for planar motion, are determined. The solutions in Laplace images are obtained by reversing the Fourier transforms. The solutions u_i (x, t), giving the longitudinal and transversal displacements are obtained by reversing the Laplace transform with the help of the development theorems and numerical methods.

Next chapter presents examples of analytical calculus for displacements fields of bars with linear viscoelastic behaviour under free vibrations, the case of the connecting rod of a parallel-crank mechanism and the case of a crank and connecting rod mechanism, where the previously presented methods are applied.

Chapter 7 deals with the central subject of the thesis. Referring to the bars with linear elastic behaviour, it is known that a state of elastic body is completely determined by the stress tensor $T_\sigma = (\sigma_{ij})$, the strains tensor $T_\varepsilon = (\varepsilon_{ij})$, i, j = 1÷3, and the displacement vector $\vec{u}$.

These three elements contain a total of 15 unknowns, which are functions of spatial coordinates x_i from elastostatics. The 15 unknowns are linked together by 3 independent groups of equations, a total of 15 equations, too:

- Equations of equilibrium: $\sigma_{ij,j} + f_i = 0; \quad i, j = \overline{1,3}$, where f_i are the projections on coordinate axes of the volumetric density of strengths acting upon elastic body;

- Equations of Cauchy (geometric equations): $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right); \quad i, j = \overline{1,3}$;

- Hooke's constitution law for the isotropic elastic body (physical equations): $\sigma_{ij} = \lambda\varepsilon\delta_{ij} + 2\mu\varepsilon_{ij}; \quad i, j = \overline{1,3}$, where ε -specific volumetric deformation: $\varepsilon = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$, λ, μ - the Lamé's parameter, $\delta_{ij} = \begin{cases} 1; i = j; \\ 0; i \neq j; \end{cases} \quad i, j = \overline{1,3}$ - Kronecker's tensor.

So, for linear elastic links subjected to vibrations, the determination of displacements fields depending on the kinematic parameters of the motion makes possible the calculus of the components of the additional strains tensor that occur due to vibrations, and then calculate the components of the additional stress tensor. Thus it can be made evident the influence of vibrations upon stress and strains states that arise in a link of a mechanism during its operation, obtaining the required data for rigorous constructive designing of the mechanism bar-type components.

For example it is considered the linear elastic connecting rod of a R(RRT) mechanism, for which the longitudinal and transversal displacements fields were previously computed by the iterative method.

In the case of bars with linear viscoelastic behaviour, it is known that linear viscoelastic bodies are comprised of two different environments, one with the properties of elastic body and one with properties of viscous fluid. In these bodies there are found instant strains growing limited or unlimited in time, a phenomenon called *creep*, and variations of tension in relation to time in the body, by maintaining a constant strain and temperature, a phenomenon called *relaxation*.

Viscoelastic solids are characterized by their ability to accumulate and distribute mechanical energy; they are part of the class of bodies with memory, their current stress state depending on the history of the suffered strains.

The complete system of equations for viscoelastic solids is written in a similar form as the one of linear elastic solid, with the difference that the functions that occur are distribution-type functions in the distributions field D'_+, depending on time $t \in R$ and on the parameter $r \in \Omega \subset R^3$ and with discontinuities of first kind in origin.

It was demonstrated that the constitutive laws of the two models of solids are in a strong dependence consisting of the fact that the Laplace or Fourier's images in distributions of the constitutive law of the solid viscoelastic coincide with the mathematical structure of the corresponding Hooke's law for the elastic solid, the complex variable "s" of the Laplace transform having the role of parameter $r \in \Omega \subset R^3$. This fact led Alfrey and Lee to the formulation of the *correspondence principle*, which may sound like this: in order to solve a viscoelasticity problem the correspondent problem from elasto-dynamics has to be solved and the Laplace image of the obtained solution has to be considered, the constants being replaced by the Laplace images of the corresponding quantities from viscoelasticity.

In technical literature it can be found determination results for the characteristics of materials with viscoelastic behaviour obtained under conditions of SR ISO 178 from 1998 relative to the determination of bending characteristics of rigid plastics using static bending test at standard temperature.

The apparent modulus of elasticity for bending as approximate value of Young's modulus, required in determining the shear modulus G and the bulk modulus K, was determined for polyvinyl chloride, using the initial linear portion of stress-strain curves. Also, the constant η of the related Newtonian component of Maxwell rheological model was determined depending on the stress at the maximum bending load σ_0 and the inclination of the line representing the area of stabilized creep. So, the Laplace transform of Young's modulus $\tilde{E}(s)$. was determined with the found values of the constants G, K and η.

It can be concluded that the displacements fields can be determined depending on the kinematical parameters of motion for viscoelastic links, too. Then, the components of the additional strains tensor that occur due to vibrations can be computed depending on the vibrations displacements fields and further, the components of the additional stress tensor. Thus it is emphasizing the influence of vibrations on the states of strains and stress that appear in a link of a mechanism, during its action. This makes possible to solve problems of links correctly sizing, eliminating the possibility of over-sizing them or their defective exploitation.

For the connecting rod of the R(RRT)mechanism, made from a viscoelastic material, the components of the strains tensor and of the stress tensor are determined analytically using the displacements fields previously calculated.

The methods described above are applied to the vibrations calculus for a connecting rod, part of a crank and connecting rod mechanism, when it is made of OLC 45, STAS 880-88, and when the connecting rod is made of un-masticated polyvinyl chloride, PVC-U (EN ISO 12608: 2003).

Polyvinyl chloride is one of the standard plastics today, with polyethylene (PE), polypropylene (PP) and polystynol (PS). Obtained from carbon, hydrogen and chlorine, polyvinyl chloride, thanks to its miscibility with various additives for improving its quality, gain a wide range of properties, which cause a wide range of specific products.

In the following paragraph, the longitudinal displacements $u_1(x,t)$ and the transversal displacements $u_2(x,t)$ due to free vibrations in the first approximation are computed using the relations from Chapter 4 and Mathematica program, for the connecting rod made of steel and a driving link speed of 206.5 rpm. It is determined the variation of these displacements for a point placed at 1/4 of the connecting rod length from the end of action.

The variations of vibration accelerations in the mentioned point are obtained as the second derivative in relation to time of the vibrations longitudinal displacement, $u_1(L/4,t)$, and vibrations transversal displacement, $u_2(L/4,t)$. The variation diagrams of the vibration accelerations are determined in the described manner for certain speeds of the driving link and certain locations of a point on the connecting rod made of steel, according to the performed experimental measurements.

The accelerations effective values for the transverse vibrations were determined for the entire range of speed of the driving link used in the experimental determinations with the connecting rod made of steel, based on the vibration accelerations calculus with Mathematica program and on their variation diagrams depending on the speed of the driving link of the type shown, in order to compare the computed values with the measured ones.

It can be noticed that the calculated (theoretical) values of the vibration acceleration on the vertical direction increases with an increasing action frequency or speed of the driving link and the highest values are recorded in the point placed at 1/4 of the length of the connecting rod measured from the end of action.

Then, the longitudinal displacements $u_1(x,t)$ and the transversal displacements $u_2(x,t)$ in the first approximation are calculated with Mathematica program for the connecting rod made of un-masticated polyvinyl chloride for different speeds between 100 and 285 rpm, and the correspondent variations in time diagrams are obtained. The variation of these displacements is determined in a point placed at 1/4 of the length of the connecting rod measured from the end of action.

The variations of vibration accelerations in the mentioned point are obtained as the second derivative in relation to time of the vibrations longitudinal displacement, $u_1(L/4,t)$, and vibrations transversal displacement, $u_2(L/4,t)$. The accelerations effective values for the transverse vibrations were determined for the entire range of speed of the driving link used in the experimental determinations with the connecting rod made of un-masticated polyvinyl chloride, based on the vibration accelerations calculus with Mathematica program and on their variation diagrams depending on the speed of the driving link of the type shown, in order to compare the computed values with the measured ones.

It can be noticed that the calculated (theoretical) values of the vibration acceleration on the vertical direction increases with an increasing action frequency or speed of the driving link and the highest values are recorded in the point placed at 1/4 of the length of the connecting rod measured from the end of action for low speeds and in the point placed in the middle of the connecting rod for higher speeds.

In order to compare the vibrations displacements of the linear elastic links with the ones of linear viscoelastic links, there are considered three speeds of driving link for which it is computed the vibrations transversal displacements for both connecting rods, the one made of steel and the viscoelastic one made of un-masticated polyvinyl chloride, respectively a minimum speed, an average speed and a maximum speed of the driving link. There are not computed the vibrations longitudinal displacements because their values are insignificant (of the order of $10^{-10} \div 10^{-7}$). For both connecting rods, the effective values of the displacements in the point placed in their middle $u_2(L/2,t)$ are computed for their comparison. It can be noticed that the computed (theoretical) values of the vibrations transversal displacements are greater for the viscoelastic connecting rod made of un-masticated polyvinyl chloride, PVC-U.

If, however, there are required for vibrations transversal displacements in the case of using the linear viscoelastic materials which does not exceed those of linear elastic metallic materials, this can be obtained increasing the cross section dimensions of the viscoelastic connecting rod.

The materials with viscoelastic behaviour are more advantageous in terms of comparable stiffness, both in terms of cost and because they have specific masses considerably lower in comparison with metallic materials, which is why the forces and moments of inertia are smaller.

The determination of stress and strains states due to free vibrations of the connecting rod of the mechanism described in paragraph 8.1, is realized with the relations for the first approximation presented in Chapter 7, first for the connecting rod made of steel and then for the connecting rod made of un-masticated polyvinyl chloride. The elements of the strains and stress tensors are computed with Mathematica program using the vibrations longitudinal displacements $u_1(x,t)$ and the vibrations transversal displacements $u_2(x,t)$ in the first approximation and their variation diagrams corresponding to the three speeds of the driving link chosen in the previous paragraph are represented.

The variation in time diagrams for the elements of the strains and stress tensors were also represented for the point placed at 1/4 of the length of the connecting rod from the end of action.

It can be noticed that the elements of the two tensors have an increasing evolution with the increasing of the driving link speed for the both connecting rods. It can be also noticed that the elements of the two tensors record minimum values in the middle of the connecting rods, excepting the component ε_{11} whose values are maximum in this area(not significant, of the order of 10^{-8}).

In order to compare the stress and strains states of the connecting rod with linear elastic behaviour, subjected to free vibrations, with the ones of the viscoelastic connecting rod, there were first determined the effective values of the elements of the two tensors whose variation diagrams were presented in the previous paragraphs corresponding to the point placed at 1/4 of the length of the connecting rods from the end of action. The equivalent stress σ_{ech} was computed using the strength of materials fifth theory, the recommended theory for the elasto-plastic domain, using the effective values of the main stresses σ_1 and σ_2.

Regarding the strains, it can be noticed that they are higher for the connecting rod made of PVC-U compared to that of steel, approximately 4 times greater at the lower action frequency and about 5-7 times greater at the highest action frequency, a fact that is explained by the lower stiffness of the viscoelastic material.

The elements of the stress tensor are higher for the connecting rod made of steel than for the one made of PVC-U. The equivalent stress is 16 - 17 times higher for the connecting rod made of steel than for the one made of PVC-U at the lower action frequency and about 9 times at the highest action frequency, due to the higher forces of inertia that appear in mechanism, when the connecting rod is made of a material with higher specific weight.

Concerning the modal analysis of the mechanism, the elements composing the crank and the connecting rod mechanism were first modeled in SolidWorks environment, then the assembly of the mechanism was modeled and it was performed the modal analysis using the module "visualNASTRAN INSIDE SolidWorks", application dedicated for numerical modeling with finite element in integrated designing. The module "visualNASTRAN INSIDE" associated to SolidWorks Program allows the use of finite element analysis with the latest capabilities of MSC/NASTRAN to automatically simulate the behaviour of parts and assemblies modeled in SolidWorks. Modal analysis are presented in the thesis in appendices for the connecting rod made of OLC 45 and the one made of PVC-U, the operation frequency being f = 4.321 Hz.

Data presented by the modal analysis confirms the conclusions of the paragraph 8.7, namely:

- Concerning the motion of the mesh nodes, it can be remarked that the maximum value of these displacements is greater for the connecting rod made of PVC-U compared to that of steel, about 6 times at the highest action frequency, which is explained by the lower stiffness of the vascoelastic material;
- The equivalent stress is about 9 times higher for the connecting rod made of steel compared to that of PVC-U, due to the higher forces of inertia that appear in mechanism, when the connecting rod is made of a material with higher specific weight.

The purpose of the experiments was to check the influence of vibrations on the dynamics of a R(RRT) mechanism and remark the differences that appear between a connecting rod with a linear elastic behaviour and a connecting rod with a linear viscoelastic behavior.

It is used an experimental assembly and there were experimentally determined the vibrations transversal accelerations in three particular points placed on the both connecting rods.

The equipment used for the experimental determination of the dynamic response is composed of:

- Data acquisition system SPIDER 8;
- Load Amplifier Bruel & Kjaer 2635 type;
- Load Amplifier Robotron M1300 type;
- Accelerometers Bruel & Kjaer 4382 type;
- Inductive transducer for linear displacement WA300.

A mechanical system crank and connecting rod type was utilised, where it was mounted successively a 11x11x1000 connecting rod, made of two types of materials, steel OLC STAS 880-88 and un-masticated polyvinyl chloride PVC-U (EN ISO 12608: 2003), and it was determined their vibrations response for variable action frequency between 1.5 and 5 Hz.

The tests were accomplished in the Machine Parts Laboratory of the Faculty of Mechanics from Craiova. An electric engine with a speed variator was used to operate and the driving was done with a trapezoidal belt. The mechanism was set on a table with T-slots, placed on a rigid frame. The engine was fixed on this frame, too, in order to make the assembly as rigid as possible for eliminating other types of vibrations. The recordings were made with a sampling frequency of 4800 samples/second for during 35 ... 40 s.

It was made a "PrelExp" program under TestPoint programming environment for determining the vibrations response of the connecting rod in time and frequency.

Conclusions:

- Errors that occur between the theoretical values of the vibrations transverse acceleration obtained after solving the mathematical model and the experimental values are less than 9.3%.

- Spectral analysis showed the following:

1. At the end of action, both connecting rods have a similar behaviour. Spectral composition is similar. Small differences occur at higher action frequencies. Spectral components of high frequency occur for the connecting rod made of steel and the harmonica of the second order occurs, too. The spectral components of high frequency are reduced for the connecting rod made of PVC-U and, besides the fundamental, the first order harmonica occurs.
2. There are some differences between the two types of connecting rods in the point of measuring placed in their middle. The spectral components of superior frequency occur at the connecting rod made of steel and the harmonics of second and fourth order are amplified. The spectral components of superior frequency are reduced at the connecting rod made of PVC-U and, besides the fundamental, the first order harmonica occurs with priority. Compared with the connecting rod made of steel, here the fundamental and the first order harmonica have greater amplitude, leading vibrations with higher effective value. For both connecting rods the transversal- horizontal oscillations respect the mentioned observations, but their amplitude is much smaller.
3. A relatively similar spectral composition occurs for both connecting rods in the point placed at 3/4 of the connecting rod length from the action end.

- It was observed that the elements of the stress and strains tensor have an increasing evolution when the driving element speed increase;

- Strains are higher for the connecting rod made of PVC-U, which is explained by the lower stiffness of the viscoelastic material;

- The elements of the stress tensor are considerably higher for the connecting rod made of steel, due to the higher forces of inertia that appear when the connecting rod is made of a material with higher specific weight.

- The paper underlines the influence of kinematic parameters on the vibrations displacements fields and implicit on the additional stress and strains states that occur due to vibrations in bar-type links, made of elastic or viscoelastic material.

Future research directions are given by:

- Experimental tests for the viscoelastic links at higher speeds than those used here;
- Study of the viscoelastic link response for technological loads in comparison with the one of the elastic link in the same imposed operating conditions;
- Study of the vibration response of a bar-type viscoelastic link with variable section, in spatial or planar motion;
- Use of more efficient programs to solve the mathematical model, to avoid approximations;

- Developing mathematical models for the vibrations of curved bars with viscoelastic behaviour in spatial or planar motion and determining methods of solving them;
- Analytical solving of models using boundary conditions other than in bars double articulated;
- Determining the stress and strains states depending on the longitudinal and transversal displacements fields, as a result of vibrations of straight or curved bars with viscoelastic behaviour as constitutive elements of spatial mechanisms.

The improvement of the solving methods of the mathematical models for viscoelastic links will contribute to the determination of the modes of vibrations for linear viscoelastic bars and will allow the observation of their behaviour in operation.

The displacements fields computing by increasing accuracy also for the viscoelastic links will be the support for future determinations of stress and strains states, useful in mechanisms designing. This will open new directions of research in dynamics of rheological solids.