UNIVERSITY OF CRAIOVA
FACULTY OF PHYSICS

ELENA-MIRELA BĂBĂLÎC

“Symmetries, supersymmetries and cohomologies in gauge theories”

–Summary of Ph.D. thesis–

Ph.D. Supervisor
Prof. Dr. SOLANGE-ODILE SALIU

1 Main subjects approached in the thesis. Method and working hypotheses

The main subjects approached in the thesis are the following: a) the derivation of the interactions in two space-time dimensions in a particular class of topological BF models; b) the construction of the couplings in $D \geq 5$ dimensions between one massless tensor field with the mixed symmetry $(3,1)$ and one with the mixed symmetry of the Riemann tensor; c) the evaluation of the existence of interactions in $D \geq 5$ dimensions between two different collections of massless tensor fields with the mixed symmetries $(3,1)$ and $(2,2)$; d) the analysis of the relation between the BRST charges obtained in the pure-spinor formalism, respectively in the κ-symmetric one for the supermembrane in eleven dimensions.

Our procedure for the first three subjects is based on solving the equations that describe the deformation of the solution to the master equation [1] by means of specific cohomological techniques [2]–[4], while for the fourth one we will use techniques specific to the BRST Hamiltonian approach in order to write the BRST charge.

The interactions are obtained under the following hypotheses: locality, Lorentz covariance, Poincaré invariance, analyticity of the deformations, and preservation of the number of derivatives on each field. The first three assumptions imply that the interacting theory is local in space-time, Lorentz covariant and Poincaré invariant. The analyticity of the deformations refers to the fact that the deformed solution to the master equation is analytical in the coupling constant and reduces to the original solution in the free limit. The conservation of the number of derivatives on each field with respect to the free theory means here that the following two requirements are simultaneously satisfied: (i) the derivative order of the equations of motion on each field is the same for the free and respectively for the interacting theory; (ii) the maximum number of derivatives in the interaction vertices is equal to two, i.e. the maximum number of derivatives from the free Lagrangian.

2 Results

In the sequel we will briefly present the main results of the thesis.

2.1 Interactions in two space-time dimensions for a particular class of BF models

The starting point is a free Lagrangian action in $D = 2$ space-time dimensions

$$S_0[A_{\mu\lambda}, B^\lambda] = \int \varepsilon^{\mu\nu} A_{\mu\lambda} \partial_\nu B^\lambda d^2x, \tag{1}$$

where the bosonic fields $A_{\mu\lambda}$ and B^λ are a tensor field of degree two and respectively a vector field. The indices of $A_{\mu\lambda}$ have no special symmetry properties. We use the metric tensor $\sigma_{\mu\nu} =$

$\mathrm{diag}(+-) = \sigma^{\mu\nu}$ and define the Levi–Civita symbol in two dimensions $\varepsilon^{\mu\nu}$ by the convention $\varepsilon^{01} = +1$. Action (1) is invariant under the generating set of gauge transformations

$$\delta_\epsilon A_{\mu\lambda} = \partial_\mu \epsilon_\lambda, \qquad \delta_\epsilon B^\lambda = 0, \tag{2}$$

where the gauge parameters ϵ_λ are bosonic. In the case where we replace the Lorentz index λ by another type, for instance by a discrete collection index $a = \overline{1, N}$, action (1) reduces to a sum of N Abelian topological BF models. In other words, the free action (1) describes a collection of two Abelian topological BF models in $D = 2$ dimensions with a maximal field spectrum, two one-forms and two scalar fields, where the collection index is Lorentz-type.

The generating set of gauge transformations (2) for the free action is irreducible and the associated gauge algebra is Abelian. The field equations corresponding to action (1) are linear in the fields and the gauge generators are field independent, so the free model under study is a linear gauge theory of Cauchy order equal to two.

The basic results on the cohomological approach to the consistent interactions that can be added to this free model can be formulated as:

A) Under the above mentioned hypotheses, we infer the Lagrangian action for the interacting theory

$$\bar{S}_0\left[A_{\mu\lambda}, B^\lambda\right] = \int \left[\varepsilon^{\mu\nu} A_{\mu\lambda}\left(\partial_\nu B^\lambda + \frac{1}{2} g W^{\lambda\rho} A_{\nu\rho}\right)\right] d^2x, \tag{3}$$

where the antisymmetric two-tensor $W^{\lambda\rho}$ must satisfy the relations

$$W^{\alpha[\sigma} \frac{\delta W^{\lambda\rho]}}{\delta B^\alpha} = 0; \tag{4}$$

B) Relations (4) admit the solution

$$W^{\lambda\rho}\left(B^\mu\right) = \varepsilon^{\lambda\rho} W\left(B^\mu\right), \tag{5}$$

where $W\left(B^\mu\right)$ is an arbitrary, smooth, scalar function that involves only the undifferentiated vector field B^μ. Under these conditions, the tensor $W^{\lambda\rho}$ gains a geometrical interpretation, namely, it represents the Poisson two-tensor associated with a Poisson manifold locally parameterized by the vector co-ordinates $\left\{B^\lambda\right\}$;

C) The cross-interaction terms

$$\frac{1}{2} g \varepsilon^{\mu\nu} A_{\mu\lambda} W^{\lambda\rho} A_{\nu\rho} \tag{6}$$

are quadratic in the tensor field of degree two from the collection, linear in the Poisson two-tensor, and break the PT invariance of the coupled action;

D) Action (3) is invariant under the deformed gauge transformations

$$\bar{\delta}_\epsilon A_{\mu\nu} = \partial_\mu \epsilon_\nu + g \frac{\delta W^{\lambda\rho}}{\delta B^\nu} A_{\mu\lambda} \epsilon_\rho, \qquad \bar{\delta}_\epsilon B^\lambda = -g W^{\lambda\rho} \epsilon_\rho; \tag{7}$$

E) The algebra of the gauge transformations (7) is open, and not Abelian, like in the free limit;

F) The gauge transformations (7) are irreducible, just like the free ones.

2.2 Interactions between one massless tensor field with the mixed symmetry $(3,1)$ and one with the mixed symmetry of the Riemann tensor

We start from the free Lagrangian action in $D \geq 5$ space-time dimensions

$$S_0\left[t_{\lambda\mu\nu|\alpha}, r_{\mu\nu|\alpha\beta}\right] = S_0^{\mathrm{t}}\left[t_{\lambda\mu\nu|\alpha}\right] + S_0^{\mathrm{r}}\left[r_{\mu\nu|\alpha\beta}\right], \tag{8}$$

where

$$\begin{aligned} S_0^{\mathrm{t}}\left[t_{\lambda\mu\nu|\alpha}\right] = & \int\left\{\frac{1}{2}\left[\left(\partial^\rho t^{\lambda\mu\nu|\alpha}\right)\left(\partial_\rho t_{\lambda\mu\nu|\alpha}\right) - \left(\partial_\alpha t^{\lambda\mu\nu|\alpha}\right)\left(\partial^\beta t_{\lambda\mu\nu|\beta}\right)\right]\right. \\ & -\frac{3}{2}\left[\left(\partial_\lambda t^{\lambda\mu\nu|\alpha}\right)\left(\partial^\rho t_{\rho\mu\nu|\alpha}\right) + \left(\partial^\rho t^{\lambda\mu}\right)\left(\partial_\rho t_{\lambda\mu}\right)\right] \\ & \left. +3\left(\partial_\alpha t^{\lambda\mu\nu|\alpha}\right)\left(\partial_\lambda t_{\mu\nu}\right) + 3\left(\partial_\rho t^{\rho\mu}\right)\left(\partial^\lambda t_{\lambda\mu}\right)\right\} d^D x, \end{aligned} \tag{9}$$

$$\begin{aligned} S_0^{\mathrm{r}}\left[r_{\mu\nu|\alpha\beta}\right] = & \int\left\{-\frac{1}{2}\left[\left(\partial_\mu r^{\mu\nu|\alpha\beta}\right)\left(\partial^\lambda r_{\lambda\nu|\alpha\beta}\right) + \left(\partial^\lambda r^{\nu\beta}\right)\left(\partial_\lambda r_{\nu\beta}\right)\right.\right. \\ & \left. +\left(\partial_\nu r^{\nu\beta}\right)\left(\partial_\beta r\right)\right] + \frac{1}{8}\left[\left(\partial^\lambda r^{\mu\nu|\alpha\beta}\right)\left(\partial_\lambda r_{\mu\nu|\alpha\beta}\right) + \left(\partial^\lambda r\right)\left(\partial_\lambda r\right)\right] \\ & \left. -\left(\partial_\mu r^{\mu\nu|\alpha\beta}\right)\left(\partial_\beta r_{\nu\alpha}\right) + \left(\partial_\nu r^{\nu\beta}\right)\left(\partial^\lambda r_{\lambda\beta}\right)\right\} d^D x. \end{aligned} \tag{10}$$

The massless tensor field $t_{\lambda\mu\nu|\alpha}$ displays the mixed symmetry $(3,1)$ (meaning it is antisymmetric in the first three indices and satisfies the identity $t_{[\lambda\mu\nu|\alpha]} \equiv 0$). The trace of $t_{\lambda\mu\nu|\alpha}$, defined by $t_{\lambda\mu} = \sigma^{\nu\alpha} t_{\lambda\mu\nu|\alpha}$, is an antisymmetric tensor, $t_{\lambda\mu} = -t_{\mu\lambda}$. The massless tensor field $r_{\mu\nu|\alpha\beta}$ has the mixed symmetry $(2,2)$ of the linearized Riemann tensor (meaning it is separately antisymmetric in the pairs $\{\mu,\nu\}$ and $\{\alpha,\beta\}$, symmetric under their permutation $\{\mu,\nu\} \longleftrightarrow \{\alpha,\beta\}$, and satisfies the identity $r_{[\mu\nu|\alpha]\beta} \equiv 0$). Its contraction of order one is a symmetric tensor, $r_{\nu\beta} = \sigma^{\mu\alpha} r_{\mu\nu|\alpha\beta}$, while that of order two is a scalar, $r = \sigma^{\nu\beta} r_{\nu\beta} \equiv r^{\mu\nu}{}_{|\mu\nu}$. The condition $D \geq 5$ is necessary in order to ensure a non-negative number of degrees of freedom for this theory. We work with a Minkowski-flat metric tensor of 'mostly plus' signature $\sigma^{\mu\nu} = \sigma_{\mu\nu} =$diag$(- + + \cdots +)$.

A generating set of gauge transformations for action (8) can be chosen of the form

$$\delta_{\epsilon,\chi} t_{\lambda\mu\nu|\alpha} = 3\partial_\alpha \epsilon_{\lambda\mu\nu} + \partial_{[\lambda}\epsilon_{\mu\nu]\alpha} + \partial_{[\lambda}\chi_{\mu\nu]|\alpha}, \tag{11}$$

$$\delta_\xi r_{\mu\nu|\alpha\beta} = \partial_\mu \xi_{\alpha\beta|\nu} - \partial_\nu \xi_{\alpha\beta|\mu} + \partial_\alpha \xi_{\mu\nu|\beta} - \partial_\beta \xi_{\mu\nu|\alpha}. \tag{12}$$

The gauge parameter $\epsilon_{\lambda\mu\nu}$ is an arbitrary, completely antisymmetric bosonic tensor, while the gauge parameters $\chi_{\mu\nu|\alpha}$ and $\xi_{\mu\nu|\alpha}$, also bosonic and arbitrary, exhibit the mixed symmetry $(2,1)$ (are antisymmetric in their first two indices and satisfy the identities $\chi_{[\mu\nu|\alpha]} \equiv 0$ and $\xi_{[\mu\nu|\alpha]} \equiv 0$). The generating set of gauge transformations (11) and (12) is off-shell, second-order reducible and the associated gauge algebra is Abelian, such that this free model is a linear gauge theory with the Cauchy order equal to four.

The main conclusions of the cohomological analysis of consistent interactions for this theory can be synthesized in:

A) Under the working hypotheses mentioned in the introductory section, we obtain the most general action of the interacting theory

$$\begin{aligned}\bar{S}_0\left[t_{\lambda\mu\nu|\alpha}, r_{\mu\nu|\alpha\beta}\right] &= S_0 + g\int\left[r - 2t_{\lambda\mu\nu|\rho}\varepsilon^{\lambda\mu\nu\alpha\beta\gamma}\left(\partial_\sigma\partial_\alpha r_{\beta\gamma|}{}^{\sigma\rho} - \frac{1}{2}\delta_\gamma^\rho\partial^\tau\partial_\alpha r_{\beta\tau}\right)\right.\\ &\left. -g\left(5r^{\lambda\rho|[\alpha\beta,\gamma]}r_{\lambda\rho|[\alpha\beta,\gamma]} - 6r_{\lambda\rho|}{}^{[\alpha\beta,\rho]}r^{\lambda\sigma|}{}_{[\alpha\beta,\sigma]}\right)\right]d^6x, \end{aligned} \tag{13}$$

where S_0 is the Lagrangian action (8), but in $D = 6$ space-time dimensions ($D = 6$ is the only case where consistent couplings appear). Action (13) contains only mixing-component terms at order one and two in the deformation parameter (plus a cosmological term);

B) Action (13) is invariant under the deformed gauge transformations

$$\bar{\delta}_{\epsilon,\chi,\xi}t_{\lambda\mu\nu|\alpha} = 3\partial_\alpha\epsilon_{\lambda\mu\nu} + \partial_{[\lambda}\epsilon_{\mu\nu]\alpha} + \partial_{[\lambda}\chi_{\mu\nu]|\alpha} - 2g\varepsilon_{\lambda\mu\nu\rho\beta\gamma}(\partial^\rho\xi^{\beta\gamma|}{}_\alpha - \frac{1}{4}\delta^\gamma{}_\alpha\partial^{[\rho}\xi^{\beta\tau]|}{}_\tau), \tag{14}$$

$$\bar{\delta}_\xi r_{\mu\nu|\alpha\beta} = \partial_\mu\xi_{\alpha\beta|\nu} - \partial_\nu\xi_{\alpha\beta|\mu} + \partial_\alpha\xi_{\mu\nu|\beta} - \partial_\beta\xi_{\mu\nu|\alpha} = \delta_\xi r_{\mu\nu|\alpha\beta}. \tag{15}$$

This is the first situation where the gauge transformations of the tensor field with the mixed symmetry $(3,1)$ get modified during the deformation process; the tensor field with the mixed symmetry $(2,2)$ is rigid under the deformation, its gauge transformations remaining unchanged;

C) The algebra of the deformed gauge transformations (14)–(15) is Abelian, like in the free limit;

D) The generating set of deformed gauge transformations remains second-order reducible, but the structure of the first-order reducibility functions is partially modified. The structure of the second-order reducibility of the coupled model is preserved with respect to the free limit;

E) If one imposes the additional requirement of PT invariance on the deformed theory, then one eliminates all the couplings mentioned previously.

2.3 Interactions between collections of massless tensor fields with the mixed symmetries $(3,1)$ and $(2,2)$

We start from a free theory in $D \geq 5$ that describes two finite collections of free massless tensor fields with the mixed symmetries $(3,1)$ and respectively $(2,2)$, with the Lagrangian action

$$S_0\left[t^A_{\lambda\mu\nu|\alpha}, r^a_{\mu\nu|\alpha\beta}\right] = S_0^{\mathrm{t}}\left[t^A_{\lambda\mu\nu|\alpha}\right] + S_0^{\mathrm{r}}\left[r^a_{\mu\nu|\alpha\beta}\right], \tag{16}$$

where

$$\begin{aligned} S_0^{\mathrm{t}}\left[t^A_{\lambda\mu\nu|\alpha}\right] &= \int\left\{\frac{1}{2}\left[\left(\partial^\rho t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial_\rho t^A_{\lambda\mu\nu|\alpha}\right) - \left(\partial_\alpha t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial^\beta t^A_{\lambda\mu\nu|\beta}\right)\right]\right.\\ &-\frac{3}{2}\left[\left(\partial_\lambda t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial^\rho t^A_{\rho\mu\nu|\alpha}\right) + \left(\partial^\rho t_A^{\lambda\mu}\right)\left(\partial_\rho t^A_{\lambda\mu}\right)\right]\\ &\left.+3\left[\left(\partial_\alpha t_A^{\lambda\mu\nu|\alpha}\right)\left(\partial_\lambda t^A_{\mu\nu}\right) + \left(\partial_\rho t_A^{\rho\mu}\right)\left(\partial^\lambda t^A_{\lambda\mu}\right)\right]\right\}d^Dx, \end{aligned} \tag{17}$$

$$S_0^{\mathrm{r}}\left[r^a_{\mu\nu|\alpha\beta}\right] = \int\left\{-\frac{1}{2}\left[\left(\partial_\mu r_a^{\mu\nu|\alpha\beta}\right)\left(\partial^\lambda r^a_{\lambda\nu|\alpha\beta}\right) + \left(\partial^\lambda r_a^{\nu\beta}\right)\left(\partial_\lambda r^a_{\nu\beta}\right)\right.\right.$$

$$+\left(\partial_{\nu}r_{a}^{\nu\beta}\right)(\partial_{\beta}r^{a})\Big]+\frac{1}{8}\left[\left(\partial^{\lambda}r_{a}^{\mu\nu|\alpha\beta}\right)\left(\partial_{\lambda}r^{a}_{\mu\nu|\alpha\beta}\right)+\left(\partial^{\lambda}r_{a}\right)(\partial_{\lambda}r^{a})\right]$$
$$-\left(\partial_{\mu}r_{a}^{\mu\nu|\alpha\beta}\right)(\partial_{\beta}r^{a}_{\nu\alpha})+\left(\partial_{\nu}r_{a}^{\nu\beta}\right)\left(\partial^{\lambda}r^{a}_{\lambda\beta}\right)\Big\}\,d^{D}x. \tag{18}$$

A generating set of gauge transformations for action (17) can be chosen of the form

$$\begin{aligned}\delta_{\epsilon,\chi}t^{A}_{\lambda\mu\nu|\alpha} &= 3\partial_{\alpha}\epsilon^{A}_{\lambda\mu\nu}+\partial_{[\lambda}\epsilon^{A}_{\mu\nu]\alpha}+\partial_{[\lambda}\chi^{A}_{\mu\nu]|\alpha}\\ &= -3\partial_{[\lambda}\epsilon^{A}_{\mu\nu\alpha]}+4\partial_{[\lambda}\epsilon^{A}_{\mu\nu]\alpha}+\partial_{[\lambda}\chi^{A}_{\mu\nu]|\alpha},\end{aligned} \tag{19}$$

where the gauge parameters $\epsilon^{A}_{\lambda\mu\nu}$ and $\chi^{A}_{\mu\nu|\alpha}$ are bosonic, the former ones being completely antisymmetric and the latter displaying the mixed symmetry $(2,1)$. The gauge algebra associated with the gauge transformations (19) is Abelian and the generating set is off-shell, second-order reducible, such that the Cauchy order of this linear gauge theory is equal to four. Action (18) allows for a generating set of gauge transformations of the form

$$\delta_{\xi}r^{a}_{\mu\nu|\alpha\beta}=\partial_{\mu}\xi^{a}_{\alpha\beta|\nu}-\partial_{\nu}\xi^{a}_{\alpha\beta|\mu}+\partial_{\alpha}\xi^{a}_{\mu\nu|\beta}-\partial_{\beta}\xi^{a}_{\mu\nu|\alpha}, \tag{20}$$

where the bosonic gauge parameters $\xi^{a}_{\mu\nu|\alpha}$ are arbitrary tensor fields with the mixed symmetry $(2,1)$. The gauge transformations (20) are Abelian and off-shell, first-order reducible, such that the Cauchy order of this linear gauge theory is equal to three. In conclusion, the free theory (16) is a linear gauge theory with an Abelian gauge algebra and a second-order reducible generating set of gauge transformations, such that the Cauchy order of this model is equal to four.

The basic results on the cohomological construction of consistent interactions that can be added to this free model are the following:

A) Related to self-interactions, we get some no-go results for each type of tensor fields;

B) The cross-couplings between these two collections of tensor fields exist only in $D=6$ dimensions. Under the same working hypotheses as before we infer the Lagrangian action of the interacting theory

$$\begin{aligned}\bar{S}_{0}\left[t^{A}_{\lambda\mu\nu|\alpha},r^{a}_{\mu\nu|\alpha\beta}\right] &= S_{0}\left[t^{A}_{\lambda\mu\nu|\alpha},r^{a}_{\mu\nu|\alpha\beta}\right]\\ &\quad +g\int\Big[c_{a}r^{a}-2f_{a}^{A}\varepsilon^{\lambda\mu\nu\alpha\beta\gamma}t_{A\lambda\mu\nu|\rho}\left(\partial_{\sigma}\partial_{\alpha}r^{a}_{\beta\gamma|}{}^{\sigma\rho}-\frac{1}{2}\delta^{\rho}{}_{\gamma}\partial^{\tau}\partial_{\alpha}r^{a}_{\beta\tau}\right)\\ &\quad -gf_{A}^{a}f_{b}^{A}\left(5r_{a}^{\lambda\rho|[\alpha\beta,\gamma]}r^{b}_{\lambda\rho|[\alpha\beta,\gamma]}-6r_{a\lambda\rho|}{}^{[\alpha\beta,\rho]}r^{b\lambda\sigma|}{}_{[\alpha\beta,\sigma]}\right)\Big]\,d^{6}x,\end{aligned} \tag{21}$$

which contains only mixing-component terms of order one and two in the deformation parameter (plus a cosmological term $gc_{a}r^{a}$).

Apparently, it seems that (21) contains non-trivial couplings between different tensor fields from the collection $\left\{r^{a}_{\mu\nu|\kappa\beta}\right\}_{a=\overline{1,n}}$ with the mixed symmetry of the Riemann tensor

$$-g^{2}f_{A}^{a}f_{b}^{A}\left(5r_{a}^{\lambda\rho|[\kappa\beta,\gamma]}r^{b}_{\lambda\rho|[\kappa\beta,\gamma]}-6r_{a\lambda\rho|}{}^{[\kappa\beta,\rho]}r^{b\lambda\sigma|}{}_{[\kappa\beta,\sigma]}\right),\quad a\neq b. \tag{22}$$

The appearance of these cross-couplings is dictated by the properties of the matrix M of elements $M_b^a = f_A^a f_b^A$. In other words, the existence of cross-couplings among different fields with the mixed symmetry of the Riemann tensor is essentially imposed by the behavior of the metric tensor in the inner space of collection indices $a = \overline{1,n}$, $\hat{k} = (k_{ab})$. Thus, if $\hat{k}$ is positive-definite, then there appear no cross-couplings among different fields with the mixed symmetry of the Riemann tensor. On the contrary, if $\hat{k}$ is indefinite, then there are allowed cross-couplings among different fields from this collection.

C) Action (21) is invariant under the deformed gauge transformations

$$\begin{aligned}\bar{\delta}_{\epsilon,\chi,\xi} t^A_{\lambda\mu\nu|\alpha} &= 3\partial_\alpha \epsilon^A_{\lambda\mu\nu} + \partial_{[\lambda}\,\epsilon^A_{\mu\nu]\alpha} + \partial_{[\lambda}\,\chi^A_{\mu\nu]|\alpha} \\ &\quad -2g f^A_a \varepsilon_{\lambda\mu\nu\rho\beta\gamma}\left(\partial^\rho \xi^{a\beta\gamma|}{}_\alpha - \frac{1}{4}\delta^\gamma{}_\alpha \partial^{[\rho}\,\xi^{a\,\beta\tau]|}{}_\tau\right), \qquad (23)\\ \bar{\delta}_\xi r^a_{\mu\nu|\alpha\beta} &= \partial_\mu \xi^a_{\alpha\beta|\nu} - \partial_\nu \xi^a_{\alpha\beta|\mu} + \partial_\alpha \xi^a_{\mu\nu|\beta} - \partial_\beta \xi^a_{\mu\nu|\alpha} = \delta_\xi r^a_{\mu\nu|\alpha\beta}. \qquad (24)\end{aligned}$$

We notice that only the gauge transformations of the fields with the mixed symmetry $(3,1)$ are modified during the deformation process;

D) The algebra of the gauge transformations for the coupled model is not affected by the deformation process, remaining Abelian, like its free limit;

E) Only the first-order reducibility functions are partially modified. The first-order reducibility relations with respect to the fields $t^A_{\lambda\mu\nu|\alpha}$ take place everywhere in the space of field histories, like the free ones, while the first-order reducibility relations associated with the fields $r^a_{\mu\nu|\alpha\beta}$ are nothing but the original ones. The reducibility functions of order two remain the initial ones, just like the corresponding reducibility relations;

F) If one imposes the additional requirement of PT invariance on the deformed theory, then one eliminates all the couplings mentioned previously.

2.4 Relating the *kappa*-symmetric and pure-spinor versions of the supermembrane in eleven dimensions

The starting point is the BST (Bergshoff-Sezgin-Townsend) action for the supermembrane in $D = 11$ in a flat supergravity background

$$\begin{aligned} S &= -\frac{1}{2}\int \mathrm{d}^3\zeta \Big[\sqrt{-g}(g^{IJ}\Pi_I^M \Pi_{JM} - 1) + \mathrm{i}\epsilon^{IJK}(\theta\Gamma_{MN}\partial_I\theta)\big[\Pi_J^M \Pi_K^N \\ &\quad + \mathrm{i}\Pi_J^M(\theta\Gamma^N\partial_K\theta) - \frac{1}{3}(\theta\Gamma^M\partial_J\theta)(\theta\Gamma^N\partial_K\theta)\big]\Big] \\ &= \int d\tau d^2\sigma \,\{\,[P_M \Pi_0^M + e^0(P_M P^M + \Delta) + e^i \Pi_i^M P_M \\ &\quad -\frac{\mathrm{i}}{2}\epsilon^{IJK}(\theta\Gamma_{MN}\partial_I\theta)[\Pi_J^M \Pi_K^N + \mathrm{i}\Pi_J^M(\theta\Gamma^N\partial_K\theta) - \frac{1}{3}(\theta\Gamma^M\partial_J\theta)(\theta\Gamma^N\partial_K\theta)]\Big\}, \qquad (25)\end{aligned}$$

which is invariant to a local (gauge) fermionic symmetry known as the κ-symmetry.

The object of components

$$\Pi_I^M = \partial_I X^M - i\theta\Gamma^M\partial_J\theta, \qquad I,J,K = 0,1,2$$

is a superfield called supersymmetric momentum, $X^M(\tau,\sigma^i)$, with Lorentz indices $M = 0,1,...,9,11$, are the supermembrane co-ordinates in superspace, the fermionic variables $\left\{\theta^A(\tau,\sigma^i)\right\}_{A=1,...,32}$ form a Majorana spinor,

$$\Delta = \det(\Pi_i^N \Pi_{jN}), \qquad i,j = 1,2,$$

while P_M represent the conjugate momenta to X^M.

The two forms written above for the action are correlated by integrating out P_M and using the parametrization $g_{IJ} \to (\gamma_{ij}, N, N^i)$

$$\begin{aligned} g_{ij} &= \gamma_{ij}\,, \qquad g_{0i} = 2\gamma_{ij}N^j, \qquad g_{00} = -N^2 + \gamma_{ij}N^iN^j, \\ g^{ij} &= \gamma^{ij} - \frac{N^iN^j}{N^2}, \qquad g^{0i} = \frac{N^i}{N^2}, \qquad g^{00} = -\frac{1}{N^2}, \end{aligned}$$

$$g = N\sqrt{\gamma} = N\sqrt{\det\gamma_{ij}}$$

together with the identifications

$$e^0 = \frac{N}{2\sqrt{\gamma}}, \qquad e^i = -N^i.$$

After performing the canonical Hamiltonian analysis we show that the theory is subject to 32 fermionic primary constraints

$$\begin{aligned} d_A &= p_A - iP_M(\Gamma^M\theta)_A \\ &\quad -\frac{1}{2}\varepsilon^{ij}(\Gamma_{MN}\theta)_A\left[\Pi_i^M\Pi_j^N + i\Pi_i^M(\theta\Gamma^N\partial_j\theta) - \frac{1}{3}(\theta\Gamma^M\partial_i\theta)(\theta\Gamma^N\partial_j\theta)\right] \\ &\quad -\frac{1}{2}\varepsilon^{ij}(\theta\Gamma_{MN}\partial_i\theta)(\Gamma^M\theta)_A(\Pi_j^N + \frac{2i}{3}\theta\Gamma^N\partial_j\theta) \approx 0 \end{aligned} \tag{26}$$

and to 3 bosonic secondary constraints, known as the reparametrization constraints

$$T = Y_MY^M + \Delta - 2\varepsilon^{ij}\Pi_i^M(d\Gamma^M\partial_j\theta) \approx 0, \tag{27}$$

$$T_i = Y_M\Pi_i^M - d\partial_i\theta \approx 0, \tag{28}$$

with the notation

$$Y_M = P_M - i\varepsilon^{ij}(\theta\Gamma_{MN}\partial_i\theta)(\Pi_j^N + \frac{i}{2}\theta\Gamma^N\partial_j\theta).$$

The fundamental Poisson brackets

$$\begin{aligned} \{P_M(\sigma), X^N(\rho)\} &= -\delta_M^N\delta^2(\sigma-\rho), \\ \{p_A(\sigma), \theta^B(\rho)\} &= -\delta_A^B\delta^2(\sigma-\rho), \end{aligned}$$

imply the following algebra for the constraints

$$\{d_A(\sigma), d_B(\rho)\} = 2iY_M\Gamma_{AB}^M\delta^2(\sigma-\rho) + i\varepsilon^{ij}\Pi_{iM}\Pi_{jN}\Gamma_{AB}^{MN}\delta^2(\sigma-\rho), \tag{29}$$

$$\{d_A(\sigma), T(\rho)\} = \{d_A(\sigma), T_i(\rho)\} = \{T(\sigma), T_i(\rho)\} = 0, \quad (30)$$

from where we conclude that T, T_i and half of d_A are first class constraints, while the other half of d_A are second class. Because there is no simple way of covariant separating the fermionic constraints we can drop the covariance and work in light-cone gauge coordinates. After a procedure of gauge un-fixing the second class constraints to first clas constraints we can start writting down the BRST charge for this theory.

For a manifest Lorentz invariant description of the supermembrane we will use the pure spinor formalism, a method recently developed by Berkovits (for the superstring), in which the supermembrane action reads

$$\begin{aligned} S &= \int d\tau d^2\sigma \left\{ K_M \Pi_0^M - d\partial_0\theta + w\partial_0\lambda \right. \\ & -\frac{i}{2}\epsilon^{IJK}(\theta\Gamma_{MN}\partial_I\theta)\left[\Pi_J^M\Pi_K^N + i\Pi_J^M(\theta\Gamma^N\partial_K\theta) - \frac{1}{3}(\theta\Gamma^M\partial_J\theta)(\theta\Gamma^N\partial_K\theta)\right] \\ & -\frac{1}{2}[K_M K^M + M + 2\epsilon^{ij}(d\Gamma_M\partial_i\theta)\Pi_J^M + 2\epsilon^{ij}(w\Gamma_M\partial_i\lambda)\Pi_J^M \\ & \left. +4i\epsilon^{ij}(w\Gamma^M\partial_i\theta)(\lambda\Gamma_M\partial_j\theta) - 4i\epsilon^{ij}(w\partial_i\theta)(\lambda\partial_j\theta)] + e^i[K_M\Pi_i^M - d\partial_i\theta + w\partial_i\lambda] \right\}, \quad (31) \end{aligned}$$

where new variables appear, such as the bosonic ghost $\left\{\lambda^A\right\}_{A=1,\ldots,32}$, which is a pure-spinor (i.e. satisfies $\lambda\Gamma^M\lambda = 0$), and its canonical conjugate momenta $\{w_A\}_{A=1,\ldots,32}$, with the gauge invariance $\delta w_A = \Lambda_M(\Gamma^M\lambda)_A$ induced by the constraint imposed on λ. A pure spinor in eleven dimensions has 23 independent components.

The "BRST charge" proposed in this formalism is

$$\mathbf{Q} = \int d^2\sigma \lambda^A d_A. \quad (32)$$

The **main conclusions** referring to the relationship between the κ-symmetric and pure-spinor versions of the supermembrane in eleven dimensions are the following:

A) It is possible to reinstate the reparametrisation constraints in the pure-spinor formulation of the supermembrane by introducing a topological sector and performing a similarity transformation. The resulting BRST charge is then of conventional type and is argued to be related to the BRST charge of the κ-symmetric supermembrane in a formulation where all second class constraints are 'gauge-unfixed' to first class constraints.

B) We encountered a natural candidate for a (non-covariant) supermembrane analogue of the superstring b-ghost, of the form

$$\begin{aligned} R &= \int d^2\sigma \left[-\frac{i}{2}K_M(d\Gamma^M\xi) + \frac{i}{4}\epsilon^{ij}\Pi_{iM}\Pi_{jN}(d\Gamma^{MN}\xi) \right. \\ & -\frac{1}{2}\epsilon^{ij}\Pi_i^M(\xi\Gamma_M\partial_j\theta)(w\lambda) - \frac{1}{4}\epsilon^{ij}\Pi_i^M(\xi\Gamma_{MNR}\partial_j\theta)(w\Gamma^{NR}\lambda) \\ & \left. -\frac{1}{2}\epsilon^{ij}\Pi_i^M(\xi\partial_j\theta)(w\Gamma_M\lambda) - \frac{1}{4}\epsilon^{ij}\Pi_i^M(\xi\Gamma^{NR}\partial_j\theta)(w\Gamma_{MNR}\lambda) \right], \quad (33) \end{aligned}$$

which satisfies

$$\{\mathbf{Q}, R\} = \int d^2\sigma[(\lambda\xi)\mathcal{T} - 2(\lambda\Gamma_M\xi)\epsilon^{ij}\Pi_i^M\mathcal{T}_j], \tag{34}$$

where $\mathcal{T}$ and $\mathcal{T}_i$ represent some ghost completions of T and T_i (27)–(28), and for $\lambda\Gamma_M\xi = 0$ and $\lambda\xi = 1$ it satisfies $\{\mathbf{Q}, R\} = T$, which represents exactly the b-ghost condition satisfied in the superstring case;

C) A strong argument in favour of (33) has to do with its behavior under the reduction procedure to the type IIA superstring in ten dimensions,

$$\Pi_2^M = \delta_{11}^M, \qquad \partial_2\theta = 0, \qquad K_{11} = \Lambda_{11} = 0, \tag{35}$$

in which case we obtain exactly the form for the superstring b-ghost known from the literature.

2.5 Published papers

The main results of this Ph.D. thesis are published in Refs. [8]–[12].

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