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"Teorii reductibile de clasa II"

-Rezumatul tezei de doctorat-

Conducator stiintific
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1 Teorii reductibile de clasa II

In teza este analizata problema abordarii ireductibile a teoriilor supuse numai la constrangeri de clasa II reductibile de un ordin arbitrar, dar finit. Abordarea ireductibila a teoriilor supuse la constrangeri de clasa II reductibile de un ordin arbitrar implica parcurgerea urmatorilor pasi:

- exprimarea parantezei Dirac pentru un sistem reductibil de clasa II in termenii unei matrici inversabile;
- constructia, intr-un spatiu al fazelor largit, a unui sistem ireductibil de clasa II echivalent cu cel reductibil original;
- deducerea parantezei Dirac pentru sistemul ireductibil obtinut anterior;
- demonstrarea egalitatii (slabe) dintre paranteza Dirac a sistemului reductibil original si cea a sistemului ireductibil asociat;
- aplicarea rezultatelor obtinute pe modele de interes fizic.

Pentru o intelegere corespunzatoare a problemei analizate si a metodei utilizate, initial au fost investigate cazurile de reductibilitate $L = 2$ si $L = 3$. Ulterior, a fost dezvoltat cazul general al unui ordin de reductibilitate arbitrar dar finit.

1.1 Abordarea ireductibila a constrangerilor de clasa II reductibile de ordinul doi

1.1.1 Constrangeri de clasa II reductibile de ordinul doi

Punctul de start il constituie un sistem descris local de N perechi canonice $z^a = (q^i, p_i)$ supus la constrangerile

$$\chi_{\alpha_0}(z^a) \approx 0,\ \alpha_0 = \overline{1, M_0}. \tag{1}$$

Presupunem ca functiile χ_{α_0} nu sunt toate independente, ci exista niste functii nenule $Z^{\alpha_0}_{\alpha_1}$ si $Z^{\alpha_1}_{\alpha_2}$ astfel incat au loc relatiile

$$Z^{\alpha_0}_{\alpha_1}\chi_{\alpha_0} = 0,\ \alpha_1 = \overline{1, M_1}, \tag{2}$$

$$Z^{\alpha_1}_{\alpha_2}Z^{\alpha_0}_{\alpha_1} \approx 0,\ \alpha_2 = \overline{1, M_2}, \tag{3}$$

cu functiile $Z^{\alpha_1}_{\alpha_2}$ independente.

Constrangerile (1) se numesc constrangeri de clasa II, daca orice subset maximal de $M \equiv M_0 - M_1 + M_2$ functii independente χ_A $(A = 1, \cdots, M)$ alese din setul χ_{α_0} ne conduce la faptul ca matricea

$$C^{(2)}_{AB} = [\chi_A, \chi_B], \tag{4}$$

este inversabila.

In termenii constrangerilor independente, paranteza Dirac se scrie

$$[F,G]^{(2)*}=[F,G]-[F,\chi_A]\,M^{(2)AB}\,[\chi_B,G]\,, \tag{5}$$

unde $M^{(2)AB}C^{(2)}_{BC}\approx\delta^A_C$.

Notam matricea parantezelor Poisson dintre functiile constrangerilor de clasa II cu

$$C^{(2)}_{\alpha_0\beta_0}=[\chi_{\alpha_0},\chi_{\beta_0}]. \tag{6}$$

Matricea $C^{(2)}_{\alpha_0\beta_0}$ nu este inveritibila

$$Z^{\alpha_0}_{\alpha_1}C^{(2)}_{\alpha_0\beta_0}\approx 0. \tag{7}$$

Fie $\bar{A}^{\alpha_1}_{\alpha_0}$ niste functii care satisfac conditia

$$rang\left(Z^{\alpha_0}_{\alpha_1}\bar{A}^{\beta_1}_{\alpha_0}\right)\equiv rang\left(D^{\beta_1}_{\alpha_1}\right)=M_1-M_2, \tag{8}$$

introducem matricea $M^{\alpha_0\beta_0}$ prin intermediul relatiei

$$C^{(2)}_{\alpha_0\gamma_0}M^{(2)\gamma_0\beta_0}\approx D^{\ \beta_0}_{\alpha_0}\equiv\delta^{\ \beta_0}_{\alpha_0}-\bar{A}^{\beta_1}_{\alpha_0}Z^{\ \beta_0}_{\beta_1}, \tag{9}$$

cu $M^{(2)\alpha_0\beta_0}=-M^{(2)\beta_0\alpha_0}$.

Structura

$$[F,G]^{(2)*}=[F,G]-[F,\chi_{\alpha_0}]\,M^{(2)\alpha_0\beta_0}\,[\chi_{\beta_0},G]\,, \tag{10}$$

defineste aceeasi paranteza Dirac ca si (5) pe suprafata (1).

Pentru un set de constrangeri de clasa II reductibil de ordinul doi paranteza Dirac se poate scrie in termenii unei matrice inversabile.

Teorema 1 *Exista o matrice inversabila $\mu^{(2)\alpha_0\beta_0}$ astfel incat paranteza Dirac (10) ia forma*

$$[F,G]^{(2)*}=[F,G]-[F,\chi_{\alpha_0}]\,\mu^{(2)\alpha_0\beta_0}\,[\chi_{\beta_0},G]\,. \tag{11}$$

pe suprafata (1).

Legatura dintre matricile $M^{(2)\alpha_0\beta_0}$ si $\mu^{(2)\alpha_0\beta_0}$ este de relatia

$$M^{(2)\alpha_0\beta_0}\approx D^{\alpha_0}_{\lambda_0}\mu^{(2)\lambda_0\sigma_0}D^{\beta_0}_{\sigma_0}. \tag{12}$$

1.1.2 Sistemul intermediar

Introducem niste variabile suplimentare $(y_{\alpha_1})_{\alpha_1=1,\cdots,M_1}$ cu parantezele Poisson date de relatiile

$$[y_{\alpha_1},y_{\beta_1}]=\omega_{\alpha_1\beta_1}. \tag{13}$$

Consideram sistemul supus la constrangerile de clasa II reductibile de ordinul doi

$$\chi_{\alpha_0}\approx 0,\ y_{\alpha_1}\approx 0. \tag{14}$$

Paranteza Dirac (in spatiul fazelor descris de coordonatele (z^a, y_{α_1})) corespunzatoare sistemului intermediar are expresia

$$\left.[F,G]^{(2)*}\right|_{z,y} = [F,G] - [F,\chi_{\alpha_0}]\,\mu^{(2)\alpha_0\beta_0}\,[\chi_{\beta_0},G] - [F,y_{\alpha_1}]\,\omega^{\alpha_1\beta_1}\,[y_{\beta_1},G]\,. \tag{15}$$

si coincide cu paranteza Dirac (in spatiul fazelor original) scrisa in termenii matricei inversabile $\mu^{(2)\alpha_0\beta_0}$

$$\left.[F,G]^{(2)*}\right|_{z,y} \approx [F,G]^{(2)*}\,. \tag{16}$$

1.1.3 Sistemul ireductibil

Teorema 2 *Exista un set de constrangeri (pe spatiul fazelor descris de coordonatele (z^a, y_{α_1}))*

$$\tilde{\chi}_{\alpha_0} = \chi_{\alpha_0} + A^{\alpha_1}_{\alpha_0} y_{\alpha_1} \approx 0,\ \tilde{\chi}_{\alpha_2} = Z^{\alpha_1}_{\alpha_2} y_{\alpha_1} \approx 0, \tag{17}$$

cu urmatoarele proprietati

(i)

$$\tilde{\chi}_{\alpha_0} \approx 0,\ \tilde{\chi}_{\alpha_2} \approx 0 \Leftrightarrow \chi_{\alpha_0} \approx 0,\ y_{\alpha_1} \approx 0. \tag{18}$$

(ii) este de clasa II si ireductibil, adica matricea

$$C_{\Delta\Delta'} = [\tilde{\chi}_\Delta, \tilde{\chi}_{\Delta'}]\,, \tag{19}$$

este inversabila, unde $\tilde{\chi}_\Delta = (\tilde{\chi}_{\alpha_0}, \tilde{\chi}_{\alpha_2})$.

Functiile $A^{\alpha_1}_{\alpha_0}$ sunt definite prin relatia

$$\bar{A}^{\alpha_1}_{\alpha_0} = A^{\beta_1}_{\alpha_0} \hat{e}^{\alpha_1}_{\beta_1}, \tag{20}$$

unde $\hat{e}^{\alpha_1}_{\beta_1}$ sunt elementele unei matrici inversabile.

Paranteza Dirac in raport cu setul de constrangeri de clasa II ireductibil are forma concreta

$$\begin{aligned}\left.[F,G]^{(2)*}\right|_{\text{ired}} = {} & [F,G] - [F,\tilde{\chi}_{\alpha_0}]\,\mu^{(2)\alpha_0\beta_0}\,[\tilde{\chi}_{\beta_0},G] - \\ & [F,\tilde{\chi}_{\alpha_0}]\,Z^{\alpha_0}_{\gamma_1}\hat{e}^{\gamma_1}_{\sigma_1}\omega^{\sigma_1\lambda_1}A^{\tau_2}_{\lambda_1}\bar{D}^{\beta_2}_{\tau_2}\,[\tilde{\chi}_{\beta_2},G] - \\ & [F,\tilde{\chi}_{\alpha_2}]\,\bar{D}^{\alpha_2}_{\lambda_2}A^{\lambda_2}_{\sigma_1}\omega^{\sigma_1\lambda_1}\hat{e}^{\gamma_1}_{\lambda_1}Z^{\beta_0}_{\gamma_1}\,[\tilde{\chi}_{\beta_0},G] - \\ & [F,\tilde{\chi}_{\alpha_2}]\,\bar{D}^{\alpha_2}_{\lambda_2}A^{\lambda_2}_{\sigma_1}\omega^{\sigma_1\lambda_1}A^{\tau_2}_{\lambda_1}\bar{D}^{\beta_2}_{\tau_2}\,[\tilde{\chi}_{\beta_2},G]\,.\end{aligned} \tag{21}$$

Teorema 3 *Paranteza Dirac asociata setului ireductibil de constrangeri de clasa II coincide cu cea a sistemului intermediar*

$$\left.[F,G]^{(2)*}\right|_{\text{ired}} \approx \left.[F,G]^{(2)*}\right|_{z,y}. \tag{22}$$

Combinand rezultatele (16) si (22) obtinem

$$[F,G]^{(2)*} \approx \left.[F,G]^{(2)*}\right|_{\text{ired}}. \tag{23}$$

1.2 Generalizare la un ordin de reductibilitate arbitrar, L

1.2.1 Constrangeri de clasa II reductibile de un ordin arbitrar, L

Consideram un sistem de constrangeri de clasa II reductibile de un ordin arbitrar, L

$$Z^{\alpha_0}_{\alpha_1}\chi_{\alpha_0}=0, \qquad Z^{\alpha_1}_{\alpha_2}Z^{\alpha_0}_{\alpha_1}\approx 0,\ldots, \qquad Z^{\alpha_{L-1}}_{\alpha_L}Z^{\alpha_{L-2}}_{\alpha_{L-1}}\approx 0, \tag{24}$$

cu $\alpha_k=\overline{1,M_k}$ pentru fiecare $k=\overline{1,L}$. Presupunem ca functiile de reductibilitate de ordin maxim (L), $Z^{\alpha_{L-1}}_{\alpha_L}$, sunt toate independente. Numarul de constrangeri de clasa II independente va fi egal cu $M\equiv\sum\limits_{k=0}^{L}(-)^k M_k$.

Paranteza Dirac in termenii a M functii independente χ_A, se scrie sub forma

$$[F,G]^{(L)*}=[F,G]-[F,\chi_A]\,M^{(L)AB}\,[\chi_B,G]\,, \qquad A=\overline{1,M}, \tag{25}$$

unde $C^{(L)}_{AB}M^{(L)BC}\approx\delta^C_A$, cu $C^{(L)}_{AB}=[\chi_A,\chi_B]$.

Matricea parantezelor Poisson dintre functiile care definesc constrangerile

$$C^{(L)}_{\alpha_0\beta_0}=[\chi_{\alpha_0},\chi_{\beta_0}] \tag{26}$$

nu este inversabila datorita relatiilor $Z^{\alpha_0}_{\alpha_1}C^{(L)}_{\alpha_0\beta_0}\approx 0$ avand rangul egal cu M.

Fie $\left(\bar{A}^{\alpha_k}_{\alpha_{k-1}}\right)_{k=\overline{1,L}}$ niste functii care satisfac relatiile

$$\operatorname{rang}\left(Z^{\beta_{k-1}}_{\alpha_k}\bar{A}^{\gamma_k}_{\beta_{k-1}}\right)\equiv\operatorname{rang}\left(D^{\gamma_k}_{\alpha_k}\right)\approx\sum_{i=k}^{L}(-)^{k+i}M_i, \tag{27}$$

$$\bar{A}^{\alpha_{k-1}}_{\alpha_{k-2}}\bar{A}^{\alpha_k}_{\alpha_{k-1}}\approx 0. \tag{28}$$

Introducem o matrice antisimetrica, de elemente $M^{(L)\alpha_0\beta_0}$, prin relatia

$$C^{(L)}_{\alpha_0\beta_0}M^{(L)\beta_0\gamma_0}\approx D^{\gamma_0}_{\alpha_0}\equiv\delta_{\alpha_0}^{\ \beta_0}-\bar{A}^{\beta_1}_{\alpha_0}Z_{\beta_1}^{\ \beta_0}, \tag{29}$$

astfel incat

$$[F,G]^{(L)*}=[F,G]-[F,\chi_{\alpha_0}]\,M^{(L)\alpha_0\beta_0}\,[\chi_{\beta_0},G] \tag{30}$$

defineste aceeasi paranteza Dirac ca si (25) pe suprafata (1).

Paranteza Dirac pentru constrangerile de clasa II reductibile de ordinul L poate fi exprimata in termenii unei matrici inversabile.

Teorema 4 *Exista o matrice inversabila, antisimetrica, $\mu^{(L)\alpha_0\beta_0}$, astfel incat paranteza Dirac (30) ia forma*

$$[F,G]^{(L)*}=[F,G]-[F,\chi_{\alpha_0}]\,\mu^{(L)\alpha_0\beta_0}\,[\chi_{\beta_0},G]\,, \tag{31}$$

pe suprafata (1).

Legatura dintre matricile $M^{(L)\alpha_0\beta_0}$ si $\mu^{(L)\alpha_0\beta_0}$ este de relatia

$$M^{(L)\alpha_0\beta_0}\approx D^{\alpha_0}_{\lambda_0}\mu^{(L)\lambda_0\sigma_0}D^{\beta_0}_{\sigma_0}. \tag{32}$$

1.2.2 Sistemul intermediar

Introducem variabilele suplimentare $\left(y_{\alpha_{2k+1}}\right)_{\alpha_{2k+1}=\overline{1,M_{2k+1}}}$, cu $k=\overline{0,\left[\frac{L-1}{2}\right]}$, avand parantezele Poisson

$$\left[y_{\alpha_i},y_{\beta_j}\right]=\omega_{\alpha_i\beta_j}\delta_{ij}. \tag{33}$$

Consideram sistemul supus la constrangerile reductibile de clasa II

$$\chi_{\alpha_0}\approx 0,\qquad \left(y_{\alpha_{2k+1}}\right)_{k=\overline{0,\left[\frac{L-1}{2}\right]}}\approx 0. \tag{34}$$

Paranteza Dirac pe spatiul fazelor parametrizat local de variabilele $\left(z^a,\left(y_{\alpha_{2k+1}}\right)_{k=\overline{0,\left[\frac{L-1}{2}\right]}}\right)$ are forma

$$\begin{aligned}\left.[F,G]^{(L)*}\right|_{z,y}&=[F,G]-[F,\chi_{\alpha_0}]\,\mu^{(L)\alpha_0\beta_0}\,[\chi_{\beta_0},G]\\&-\sum_{k=0}^{\left[\frac{L-1}{2}\right]}\left[F,y_{\alpha_{2k+1}}\right]\omega^{\alpha_{2k+1}\beta_{2k+1}}\left[y_{\beta_{2k+1}},G\right],\end{aligned} \tag{35}$$

si coincide cu paranteza Dirac (in spatiul fazelor original) scrisa in termenii matricei inversabile $\mu^{(L)\alpha_0\beta_0}$

$$\left.[F,G]^{(L)*}\right|_{z,y}\approx[F,G]^{(L)*}. \tag{36}$$

1.2.3 Sistemul ireductibil

Teorema 5 *Exista un set de constrangeri (pe spatiul fazelor descris de coordonatele $\left(z^a,\left(y_{\alpha_{2k+1}}\right)_{k=\overline{0,\left[\frac{L-1}{2}\right]}}\right)$)*

-daca L este impar

$$\tilde{\chi}_{\alpha_0}\equiv\chi_{\alpha_0}+A^{\alpha_1}_{\alpha_0}y_{\alpha_1}\approx 0, \tag{37}$$

$$\tilde{\chi}_{\alpha_{2k}}\equiv Z^{\alpha_{2k-1}}_{\alpha_{2k}}y_{\alpha_{2k-1}}+A^{\alpha_{2k+1}}_{\alpha_{2k}}y_{\alpha_{2k+1}}\approx 0,\quad k=\overline{1,\left[\frac{L}{2}\right]}; \tag{38}$$

-daca L este par

$$\tilde{\chi}_{\alpha_0}\equiv\chi_{\alpha_0}+A^{\alpha_1}_{\alpha_0}y_{\alpha_1}\approx 0, \tag{39}$$

$$\tilde{\chi}_{\alpha_{2k}}\equiv Z^{\alpha_{2k-1}}_{\alpha_{2k}}y_{\alpha_{2k-1}}+A^{\alpha_{2k+1}}_{\alpha_{2k}}y_{\alpha_{2k+1}}\approx 0,\quad k=\overline{1,\frac{L}{2}-1}, \tag{40}$$

$$\tilde{\chi}_{\alpha_L}\equiv Z^{\alpha_{L-1}}_{\alpha_L}y_{\alpha_{L-1}}\approx 0; \tag{41}$$

cu urmatoarele proprietati

(i)

$$\left(\tilde{\chi}_{\alpha_{2k}}\right)_{k=\overline{0,\left[\frac{L}{2}\right]}}\approx 0\Leftrightarrow\left(\chi_{\alpha_0}\approx 0,\left(y_{\alpha_{2k+1}}\right)_{k=\overline{0,\left[\frac{L-1}{2}\right]}}\approx 0\right); \tag{42}$$

(ii) este de clasa II si ireductibil, adica matricea de elemente

$$C_{\Delta\Delta'} = [\tilde{\chi}_{\Delta}, \tilde{\chi}_{\Delta'}] , \tag{43}$$

este inversabila, unde $\tilde{\chi}_{\Delta} \equiv (\tilde{\chi}_{\alpha_{2k}})_{k=\overline{0,[\frac{L}{2}]}}$.

Functiile $A_{\alpha_{2k}}^{\alpha_{2k+1}}$ de mai sus sunt definite de relatiile:
-daca L este impar

$$\bar{A}_{\alpha_{2k}}^{\alpha_{2k+1}} = A_{\alpha_{2k}}^{\beta_{2k+1}} \hat{e}_{\beta_{2k+1}}^{\alpha_{2k+1}}, \qquad k = \overline{0, \left[\frac{L}{2}\right] - 1}, \tag{44}$$

$$\bar{A}_{\alpha_{L-1}}^{\alpha_L} = A_{\alpha_{L-1}}^{\beta_L} \bar{D}_{\beta_L}^{\alpha_L}; \tag{45}$$

-daca L este par

$$\bar{A}_{\alpha_{2k}}^{\alpha_{2k+1}} = A_{\alpha_{2k}}^{\beta_{2k+1}} \hat{e}_{\beta_{2k+1}}^{\alpha_{2k+1}}, \qquad k = \overline{0, \frac{L}{2} - 1}. \tag{46}$$

Elementele $\hat{e}_{\beta_{2k+1}}^{\alpha_{2k+1}}$ determina o matrice inversabila, iar $\bar{D}_{\beta_L}^{\alpha_L}$ este inversa lui $D_{\alpha_L}^{\beta_L} = Z_{\alpha_L}^{\gamma_{L-1}} A_{\gamma_{L-1}}^{\beta_L}$.
Paranteza Dirac in raport cu setul de constrangeri de clasa II ireductibil are forma

$$\begin{aligned}
&\left.[F, G]^{(L)*}\right|_{\mathrm{ired}} = [F, G] - [F, \tilde{\chi}_{\alpha_0}]\, \mu^{(L)\alpha_0\beta_0}\, [\tilde{\chi}_{\beta_0}, G] \\
&- \sum_{k=0}^{\left[\frac{L}{2}\right]-1} \Big\{ [F, \tilde{\chi}_{\alpha_{2k}}]\, Z_{\alpha_{2k+1}}^{\alpha_{2k}} \hat{e}_{\gamma_{2k+1}}^{\alpha_{2k+1}} \omega^{\gamma_{2k+1}\beta_{2k+1}} \bar{A}_{\beta_{2k+1}}^{\beta_{2k+2}} [\tilde{\chi}_{\beta_{2k+2}}, G] \\
&+ \left[F, \tilde{\chi}_{\alpha_{2k+2}}\right] \bar{A}_{\alpha_{2k+1}}^{\alpha_{2k+2}} \omega^{\alpha_{2k+1}\gamma_{2k+1}} \hat{e}_{\gamma_{2k+1}}^{\beta_{2k+1}} Z_{\beta_{2k+1}}^{\beta_{2k}} [\tilde{\chi}_{\beta_{2k}}, G] \\
&+ \left[F, \tilde{\chi}_{\alpha_{2k+2}}\right] \psi^{\alpha_{2k+2}\beta_{2k+2}} \left[\tilde{\chi}_{\beta_{2k+2}}, G\right] \Big\} .
\end{aligned} \tag{47}$$

Teorema 6 *Paranteza Dirac asociata setului ireductibil de constrangeri de clasa II coincide cu cea a sistemului intermediar*

$$\left.[F, G]^{(L)*}\right|_{\mathrm{ired}} \approx \left.[F, G]^{(L)*}\right|_{z,y} . \tag{48}$$

Combinand rezultatele (36) si (48) obtinem

$$[F, G]^{(L)*} \approx \left.[F, G]^{(L)*}\right|_{\mathrm{ired}} . \tag{49}$$

Rezultatele de baza ale tezei sunt continute in lucrarile:

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